Many-qubit network employing cavity QED in a decoherence-free subspace

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We propose a many-qubit network with cavity QED by encoding qubits in decoherence-free subspace, based on which we can implement many-logic-qubit conditional gates by means of cavity assisted interaction with single-photon pulses. Our scheme could not only resist collective dephasing errors, but also much reduce the implementational steps compared to conventional methods doing the same job, and we can also complete communications between two arbitrary nodes. We show the details by implementing a three-logic-qubit Toffoli gate, and explore the experimental feasibility and challenge based on currently achievable cavity QED technologies.

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In quantum information science, one-qubit rotations and two-qubit conditional operations could constitute universal quantum computing (QC) [1]. Although these basic operations have been achieved experimentally in various systems, to have a large-scale QC efficiently with high-fidelity, we are still exploring direct accomplishment of many-qubit conditional gates for simplifying the operational steps and decreasing the implementing time.

Cavity QED system has been considered as a wonderful device to realize the quantum information processing. The confined atoms in cavity QED system are not only suited for storing qubits with long-lived atomic internal states, but also usable for repeaters in quantum networks [2, 3]. We have noticed recent proposals for multi-atom quantum gates carried out in a single cavity [4] and in a network of non-local cavities [5], which provide hopeful ways for scalable QC with cavity QED.

However, decoherence due to inevitable interaction with environment always damages quantum gating. In this Brief Report, we will focus on avoiding collective dephasing errors by decoherence-free subspace (DFS) [6, 7, 8], in which some unpredictable collective dephasing due to, for example, ambient magnetic fluctuations, would be kept away from our encoding qubits. To get this benefit, we have to pay a price with two physical qubits encoding one logic qubit. Specifically, in our present case, two atoms in one cavity encode a single logic qubit [8], with the form $|\widetilde{0}\rangle \equiv |1\rangle_1|0\rangle_2 = |10\rangle$ and $|\widetilde{1}\rangle \equiv |0\rangle_1|1\rangle_2 = |01\rangle$.

With this encoding, we will propose a nonlocal many-qubit gating in DFS in a quantum network constituted by cavities, based on cavity-assisted interaction by single-photon interference. To make our description clarified, we will take the three-logic-qubit Toffoli gate as an example, and our idea is directly applicable to the case of arbitrary numbers of logic qubits. The favorable features of our scheme include a big reduction of the imple-

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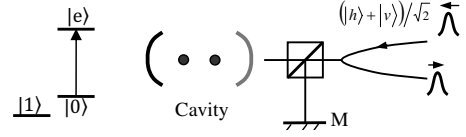†Electronic address: mangfeng@wipm.ac.cn

FIG. 1: Left: Level configuration of an atom in our scheme, where the two ground states $|0\rangle$ and $|1\rangle$ with physical qubits encoded are of large energy splitting, and $|e\rangle$ is the excited state. Right: Two identical atoms are well located in a high-finesse single-mode cavity to implement a two-atom U_{CPF} by cavity-assisted polarized-photon scattering. The polarizing beam splitter (PBS) transmits (reflects) the h (v) component of an input single-photon pulse.

mentational steps compared to conventional methods by the network of one- and two-qubit quantum gates. Besides, the Hadamard operation of the single-logic-qubit and multi-logic-qubit conditional operations could coexist in our design but work independently due to our elaborate control of the path of the single-photons, which helps for compatibility of the quantum network. Furthermore, our network allows quantum communications between arbitrary two cavity nodes.

The fundamental cavity-assisted photon scattering to realize a conditional phase flip (CPF) between two atoms in a cavity has been reiterated in [3, 5, 9]. As shown in Fig. 1, the cavity mode, the input photon pulse and the transition between the levels $|0\rangle$ and $|e\rangle$ of the atom are resonantly coupled, while level $|1\rangle$ is decoupled owing to the large detuning. Because the cavity mode is h polarized, it only interacts with the h component of an input photon. The atom in $|0\rangle$ will shift the cavity mode so that the photon pulse will leave the cavity with nothing changed. While the atom in $|1\rangle$ leads to the π phase added when the photon left because of the resonance between the photon and the cavity mode. The key point of the CPF is that an input single-photon pulse with h polarization could interact with the cavity mode if and only if the two atoms are in state $|1\rangle_1|1\rangle_2$. As a result, the single-photon pulse moving in and then out of the cavity yields a two-atom CPF oper-

ation $U_{\mathrm{CPF}} = \exp(i\pi|1\rangle_{11}\langle 1| \otimes |1\rangle_{22}\langle 1|)$.

With the cavity-assisted photon scattering, both the single-logic-qubit Hadamard gate $\widetilde{H}$ and the two-logic-qubit conditional phase gate $\widetilde{U}_{\mathrm{CP2}}$ had been realized between two neighboring nodes in a cavity-based system [9], where $\widetilde{H}$ depends on twice interactions of the photon with the cavity and $\widetilde{U}_{\mathrm{CP2}}$ needs only one interaction with each cavity. While to extend our idea to be more than two cavities, we need to design a more smart device. Fig. 2 demonstrates from different views the basic unit of such a design, where the half-wave plate 45° (HWP45°), with its axis at 45° to the horizontal direction, rotates the photon polarization as $|h\rangle \leftrightarrow |v\rangle$. HWP22.5°, at an angle of 22.5° to the horizontal direction, performs a Hadamard gate on the photon polarization states, i.e., $|h\rangle \leftrightarrow (1/\sqrt{2})(|h\rangle + |v\rangle)$, $|v\rangle \leftrightarrow (1/\sqrt{2})(|h\rangle - |v\rangle)$. C and C* are circulators. P_{45} is a 45° polarizer projecting the polarization $(|h\rangle + |v\rangle)/\sqrt{2}$. The different graphic denotations of PBS and TR are due to different views.

TR, marked in purple, including TR0, TR1, TR2, TR3, TRin and TRout in Fig. 2, are optical devices which can be controlled exactly as needed to transmit or reflect a photon with a very fast switching time. All the TR devices are identical, but labeled to be TRm ($m = 0, 1, 2, \cdots$) for convenience of our description. STR, a special device which controls the path of the photon to different planes, has three ports allowing the single-photon pulses to communicate with the cavity node. The arrows in different color in Fig. 2 show the different photon paths regarding the three ports, respectively. The single-photon pulse can enter the cavity node directly from the port 0 along the black arrow depending on the transmitting states of TRin and TRout, as shown in the top plot of Fig. 2(b). Alternatively, STR allows the photon input from the port 1 following the red thin arrow (Fig. 2(c2)). After its interacting with the atoms in the cavity, the single-photon pulse should move along the blue fat arrow going to TR2 and then be reflected out of the node from the port 2 (Fig. 2(c1)). Moreover, if it just passes by the cavity node, the single-photon pulse will pass through STR from the port 1 directly to the port 2 with the help of TR1 and TR3, as shown in Fig. 2(c3).

Within the cavity node, the movement of the single-photon pulse also depends on the state of TR0. When TR0 is switched to a transmitting state, the photon will pass through TR0 and P_{45} and then be measured by the detector directly. This is part of the way for $\widetilde{H}$ [9]. To carry out conditional gates, we require TR0 to keep in a reflecting state, which makes the photon go out of the cavity node along the blue fat arrow from the port 2. To implement our scheme, we have to switch the relevant TR into the corresponding transmitting or reflecting state according the photon paths in Fig. 2.

Based on above basic unit, we construct a circular many-qubit cavity network in DFS in Fig. 3. To make our description clarified and simple, without loss of generality, we only demonstrate a three-logic-qubit Toffoli gate $\widetilde{T}_{\mathrm{offoli}}^{ij,k} = \widetilde{H}_k \otimes \widetilde{U}_{\mathrm{CP3}} \otimes \widetilde{H}_k$ as an example, where $\widetilde{H}_k$ and $\widetilde{U}_{\mathrm{CP3}}$ are Hadamard gate on the kth logic-qubit and three-logic-qubit conditional phase gate, respectively. For the many-qubit conditional gate, the key point is not the order of the operations on individual nodes, but the phase flip when all target logic-qubits are in states $|\widetilde{1}\rangle$. So as an example, we give in Fig. 3 an implementation of $\widetilde{U}_{\mathrm{CP3}}$ by a clock-wise operations based on the single-photon interference.

The operation of $\widetilde{U}_{\mathrm{CP3}}$ as shown in Fig. 3 will follow the thick blue optical path on the circle. It can be performed by following steps. A single-photon pulse in state $(1/\sqrt{2})(|h\rangle + |v\rangle)$ is imported from the port $0i$, i.e., the port 0 belonging to the ith node. As mentioned previously in Fig. 2, the input photon will enter the node$i$ directly through STRi, and reach the cavity$i$ through C and PBS. To have the operation U_{CPF}, we perform a σ_x operation on the atom 2 in the cavity$i$ (denoted as $\sigma_{x,i}^2$) so that U_{CPF} could happen when the single-photon pulse moves in and then out of the cavity. Then we carry out another σ_x operation to restore the state of the atom 2. Subsequently, the photon moves back to C again, turned by C and then reflected by TR0 to HWP45°. Finally, the single-photon pulse moves out of the node$i$ from the port $2i$ of STRi. Let us turn to Fig. 3 again. After the single-photon pulse moves out of the node$i$, it clock-wisely arrives at PBSa through TRiI along the thick blue line. Whether the photon pulse will pass through PBSa to node$j$ or be reflected along the path$a$ to PBSd depends on the state of the logic-qubit insides the node$i$ (i.e., cavity$i$). Specifically, for the logic-qubit in cavity$i$ in state $|\widetilde{0}\rangle_i$ or $|\widetilde{1}\rangle_i$, the above photon path in the basic unit leads to

$$\frac{|h\rangle + |v\rangle}{\sqrt{2}}|\widetilde{0}\rangle_i \xrightarrow{\sigma_{x,i}^2 U_{\mathrm{CFP}} \sigma_{x,i}^2} \frac{-|h\rangle + |v\rangle}{\sqrt{2}}|\widetilde{0}\rangle_i$$
$$\xrightarrow{\mathrm{HWP45°}} \frac{|h\rangle - |v\rangle}{\sqrt{2}}|\widetilde{0}\rangle_i \xrightarrow{\mathrm{HWP22.5°}} |v\rangle|\widetilde{0}\rangle_i. \tag{1}$$

$$\frac{|h\rangle + |v\rangle}{\sqrt{2}}|\widetilde{1}\rangle_i \xrightarrow{\sigma_{x,i}^2 U_{\mathrm{CFP}} \sigma_{x,i}^2} \frac{|h\rangle + |v\rangle}{\sqrt{2}}|\widetilde{1}\rangle_i$$
$$\xrightarrow{\mathrm{HWP45°}} \frac{|h\rangle + |v\rangle}{\sqrt{2}}|\widetilde{1}\rangle_i \xrightarrow{\mathrm{HWP22.5°}} |h\rangle|\widetilde{1}\rangle_i. \tag{2}$$

So if the logic-qubit in cavity is in state $|\widetilde{0}\rangle$, the single-photon pulse will be reflected by PBSa to go along the path$a$, whereas the photon will go through PBSa and HWP22.5° to the node$j$ if the atoms are in $|\widetilde{1}\rangle$. For the latter case, the h-polarized photon will go through a HWP22.5° before entering the cavity node$j$, and thereby becomes in superposition $(1/\sqrt{2})(|h\rangle + |v\rangle)$ again. The interaction between the photon pulse and the atoms in cavity$j$ is similar to in the cavity$i$, and the subsequent process to the cavity$k$ is similar to that to the cavity$j$. In our three-logic-qubit case, the whole
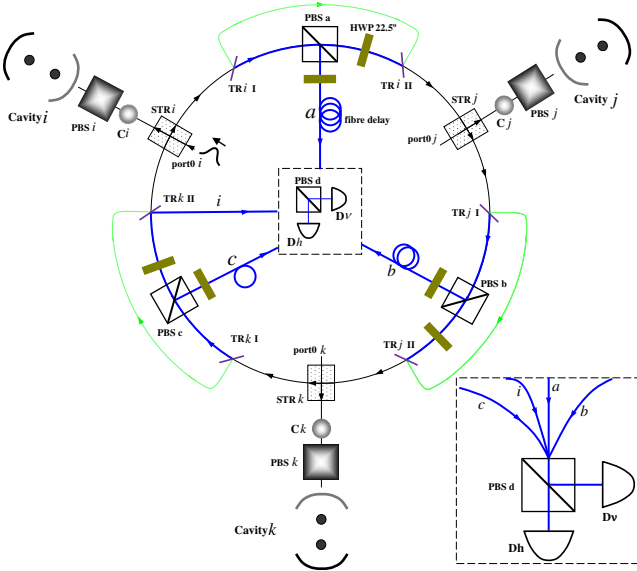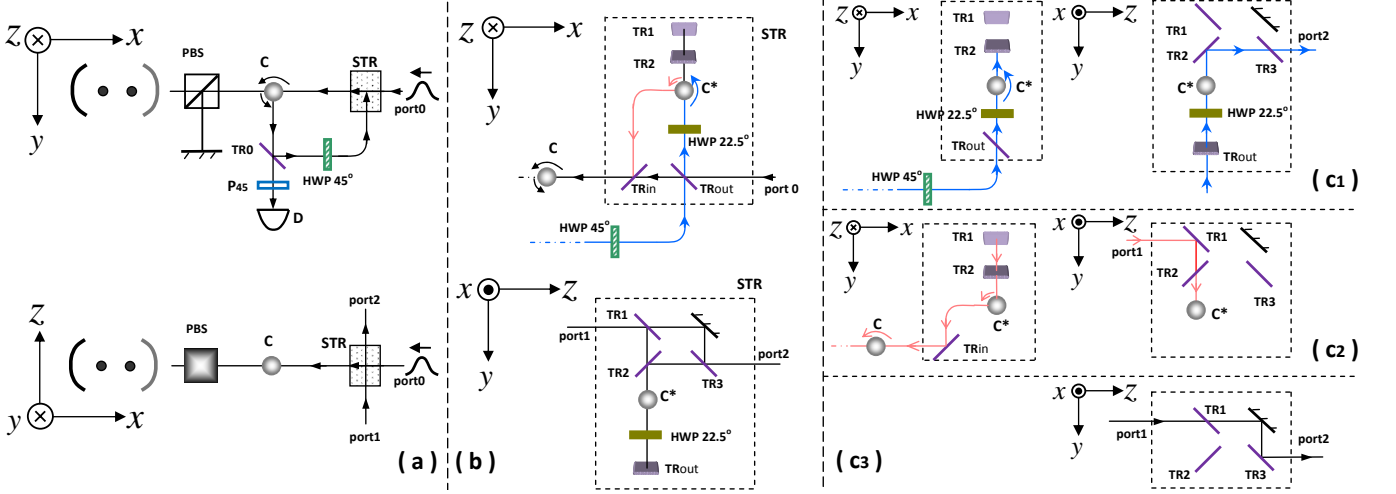
FIG. 2: (Color online) Schematic setup of the basic node unit of our design from different views. (a) Setup viewed along z-axis and viewed on the $x - z$ plane; (b) The configuration of STR, which consists of several optical elements, such as TR (marking in purple) , circulator and haft-wave plate. STR can control the path of the photon to different planes (see text for detail). The single-photon pulse can enter the cavity node directly from the port 0 (along the black arrow); (c) The photon can be guided in the cavity node from the port 1 (along the red thin arrow), and be guided out of the cavity node from the port 2 (along the blue fat arrow). Furthermore, the photon can cross STR directly between port 1 and port 2. HWP45° rotates the photon polarization as $|h\rangle \leftrightarrow |v\rangle$, HWP22.5° perform Hadamard operation on the photon state. D is a detector, C and C* are circulators, P_{45} is a 45° polarizer. TR can be controlled exactly as needed to transmit or to reflect a photon.

FIG. 3: (Color online) Schematic setup for implementation of three-logic-qubit conditional phase gate $\widetilde{U}_{\mathrm{CP3}}$ by single-photon interference in clock-wise direction. The photon will take four possible paths a, b, c and i to reach PBSd. The single-logic-qubit Hadamard operation and the two- and multi-logic-qubit conditional operation are coexisting in our scheme but could work independently by controlling the path of the single-photons. The thick blue circle is the necessary path for the $\widetilde{U}_{\mathrm{CP3}}$ operation, and the light green path for the $\widetilde{U}_{\mathrm{CP2}}$ between two logic qubits. Each dark-green half-wave plate stands for a HWP 22.5°. The inset is for the detection made at the center of the network.

optical path ends at reaching the detector Dh or Dv along the path$i$. The click of Dv yields a minus sign in front of the state $|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k$, and the click of D$h$ means nothing changed. Straightforward deduction could show that, for three logic-qubits initially in an arbitrary state $\beta_1|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k + \beta_2|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k + \beta_3|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k + \beta_4|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k + \beta_5|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k + \beta_6|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k + \beta_7|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k + \beta_8|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k$, the above process yields

$$
\frac{|h\rangle}{\sqrt{2}} \otimes (\beta_1|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k + \beta_2|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k + \beta_3|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k
$$
$$
+ \beta_4|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k + \beta_5|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k + \beta_6|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k +
$$
$$
\beta_7|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k + \beta_8|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k) - \frac{|v\rangle}{\sqrt{2}} \otimes (\beta_1|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k
$$
$$
+ \beta_2|\widetilde{0}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k + \beta_3|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k + \beta_4|\widetilde{0}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k +
$$
$$
\beta_5|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{0}\rangle_k + \beta_6|\widetilde{1}\rangle_i|\widetilde{0}\rangle_j|\widetilde{1}\rangle_k + \beta_7|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{0}\rangle_k
$$
$$
- \beta_8|\widetilde{1}\rangle_i|\widetilde{1}\rangle_j|\widetilde{1}\rangle_k).
$$
$$(3)$$

The measurement is made on the output photon by detectors Dh and Dv behind PBSd. As PBS transmits (reflects) h (v)-polarized photon, according to Eq. (3), the three-logic-qubit conditional phase gate $\widetilde{U}_{\mathrm{CP3}}$ succeeds if Dv clicks. With the help of single-logic-qubit $\widetilde{H}$ operation, we can carry out a standard Toffoli gate in DFS by $\widetilde{T}_{\mathrm{offoli}}^{ijk} = \widetilde{H}_k \otimes \widetilde{U}_{\mathrm{CP3}} \otimes \widetilde{H}_k$. It must be mentioned that the optical length of the paths a, b, c and i must be equal to suppress the phase instability in the single-photon interference.

Different operations can be distinguished from the corresponding ports and detectors. The single-logic-qubit operation $\widetilde{H}_i$ in the ith cavity is associated with a single-photon pulse input from the port $0i$ and output to the detector Di [9], whereas the many-qubit gating is related to the input from the port $0i$ and the output to the detector Dv. Since different gates could coexist in our design and work independently, our design is not only scalable but also compact.

It is evident that our scheme can be directly extended to be more than three-node case. Meanwhile, it could implement the conditional phase flip between two arbitrary nodes. For example, by switching TRi (including TRiI and TRiII) to the reflecting state, we may have the single-photon pulse skipping over PBSa. When it arrives at the STRj, the single-photon pulse can also be chosen entering node$j$ or not according to Fig. 2(c). In this case, we can implement the communications between two arbitrary nodes instead of only the neighboring nodes [9].

The currently achieved technology of deterministic single-photon source [10], with 10,000 high-quality single photons generated continuously per second, supports a fast implementation of our scheme. About the logic-qubits, we may confine the atoms in optical lattices embedded in an optical cavity which has already been achieved experimentally [11]. But current techniques have not yet enabled the atoms individually confined in some particular lattice sites. Alternatively, we may consider two charged atoms confined by a trap potential and optically coupled by the cavity mode. A single Calcium ion has been successfully trapped in such a device [12]. To achieve our scheme, however, we require the above experiment to be extended to two ions.

For the cavity-assisted operation U_{CPF}, the numerical simulations had been made in Ref.[3], which shows that, if the duration T for the photon pulse input in the cavity and the cavity decay rate κ satisfy $\kappa T \gg 1$, U_{CPF} is insensitive to both the atom-cavity coupling strength and the Lamb-Dicke localization. Specifically, if $\kappa T \sim 100$ and the atom-cavity coupling is several times stronger than the dissipative rates of the system, the gate fidelity is almost unity ($F > 99.5\%$)[3]. Therefore, with the experimental numbers $\kappa/2\pi \sim 4$ MHz, $g/2\pi \sim 30$ MHz, $\Gamma/2\pi \sim 2.6$ MHz [13, 14], we may estimate the time for U_{CPF} and $\widetilde{H}$ to be about $3 \sim 5\mu s$ and $6 \sim 10\mu s$, re-

spectively, for $\kappa T \gg 1$. The time of the $\widetilde{U}_{\mathrm{CP}N}$ operation depends on the number of the logic qubits. For example, for the case of $N =3$, 4 and 5, the gating time of $\widetilde{U}_{\mathrm{CP}N}$ would be about $\sim 12\mu s$, $\sim 16\mu s$ and $\sim 20\mu s$, respectively.

The possible imperfection is mainly from the photon loss, the phase instability, detector inefficiency, and other logic errors beyond collective dephasing in real QC operations, such as leakage errors and so on. Like the repeat-until-success scheme [15] which discards the photon loss events by photon detection, our scheme could reach high fidelity by the photon detection even in the case of photon loss. Besides, the phase stability could be guaranteed by keeping the path lengths of the photons stable at sub-wavelength levels. Moreover, the current dark count rate of the single-photon detector is about 100 Hz, which could reduce the efficiency of our scheme by a factor of 10^{-4}. But this is not an intrinsic drawback of our scheme itself. As for the logic errors beyond collective dephasing, we may suppress them by some elaborately designed pulse sequences, e.g., with 'Bang-Bang' control pulses on the encoded qubits and then amended by refocusing on individual physical qubits [16], or with some specially designed pulses [17]. To suppress these unpredictable errors, we have to mention two points below. First, we have supposed in our model that the collective dephasing errors are dominant for the atomic qubits in a cavity at very low temperature, which implies that the quantum gating with DFS employed in our scheme should behave better than others without using DFS. The second point is that both 'Bang-Bang' control and refocusing have been sophisticated techniques. So all the errors would be strongly suppressed.

In summary, we have proposed a many-logic-qubit network of cavity QED, and demonstrated a three-logic-qubit Toffoli gate in DFS, which could either carry out conditional gates between two arbitrary logic qubits, or be extended to many-logic-qubit conditional gates. We argue that our design is compact, scalable, and collective dephasing resisted, which is practical in quantum network of cavity QED.

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